

# Internal gravity wave dynamics near a critical level

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## Objectives

- Examine the effect of Reynolds number on the critical layer interaction via Spectral Multidomain Penalty Method (Diamessis et al., JCP 2005) for the solution of incompressible N.S. equations under the Boussinesq approximation.
- Study the momentum transfer to the mean flow.
- Visualize the Structure of vorticity and isopycnal fields.
- Assess the role of shear vs. convective instability in critical layer evolution.

## Numerical generation of IGW's

We use momentum source terms based on localizing the periodic solution of the linearized IGW equations (Slinn and Riley 1998).

$$F_u = \phi(t) \left\{ \frac{-Am}{k} F(z) \cos(kx + mz - \alpha t) - \frac{A}{k} F'(z) \sin(kx + mz - \alpha t) \right\}$$

$$F_w = \phi(t) \{ AF(z) \cos(kx + mz - \alpha t) \}$$

$$F_\rho = \phi(t) \left\{ \frac{-AF(z)}{\omega} F(z) \sin(kx + mz - \alpha t) \right\}$$

$$\phi(t) = \begin{cases} 1 & t < \frac{2\pi}{\omega} \\ 0 & t > \frac{2\pi}{\omega} \end{cases}, \quad F(z) = \exp(-a \left( \frac{z - z_0}{\lambda_x} \right)^2)$$

- Nonlinearity caused by the Spatial localization of the forcing terms leads to generation of a strong mean flow in the forcing region. The mean flow strongly interacts with the propagating wave leading to its weakening and the distortion of its tail.
- The zero Fourier mode is deactivated in the forcing region to prevent the formation of the mean flow.

## Summary of the results

- The background shear flow is accelerated as a result of momentum exchange with the IGW.
- At the lowest studied Re ( $1.5 \times 10^5$ ) the wave is completely absorbed at the critical level.
- At high Re ( $4.5 \times 10^5$ ,  $1.5 \times 10^6$ ) Instability sets in around the critical level, characterized by small scale structures.
- Overturning of the isopycnal surfaces persists for several buoyancy periods before the onset of the instability.
- The instability (Kelvin-Helmholtz) is driven by the intensification of the shear resulting from the shrinking of the vertical scale of the waves as they approach the critical level.
- The wave-mean flow interaction is strongest at high Re and the interaction region has a larger vertical extent.

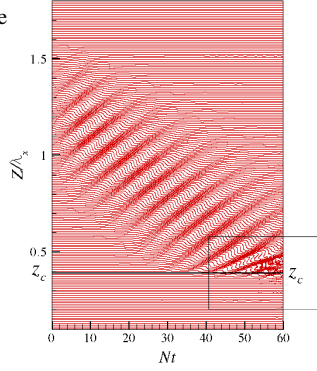


Fig.1 Depth time diagram for  $Re=1.5 \times 10^6$

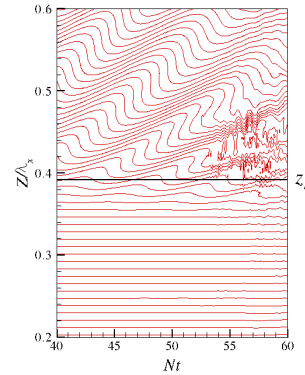


Fig.2 Close-up of the interaction region in Fig.1

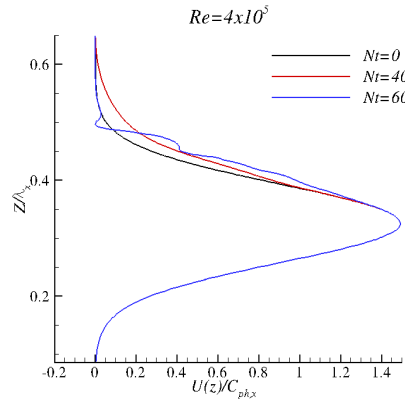


Fig.3 Background mean flow evolution during the interaction with the IGW at  $Re=4 \times 10^5$

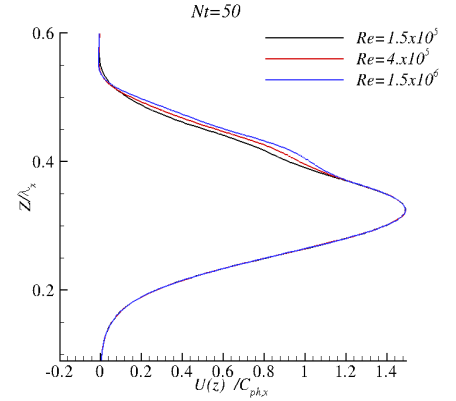


Fig.4 Re-dependence of background velocity profile immediately prior to the onset of instability

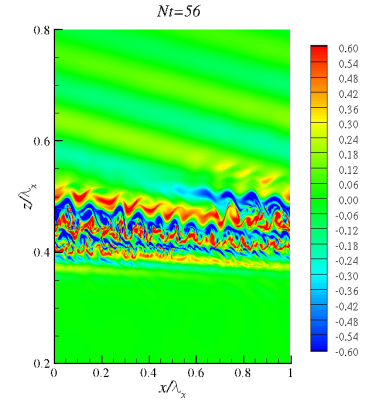


Fig.5 Vorticity field after the onset of Kelvin-Helmholtz instability

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## References

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